

Exam 2-Math 210

1. (20 points) Use the ϵ - δ definition to show that

$$\lim_{x \rightarrow 3} x^2 + 2x + 1 = 16.$$

2. (20 points) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases} .$$

- (a) Show that $f(x)$ has a limit at $x = 0$
- (b) Show that f does not have a limit at c for $c \neq 0$.

3. (a) (20 points) Let

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases} .$$

Show that $f(x)$ is discontinuous everywhere in $[0,1]$.

(b) Give a function $f : [0, 1] \rightarrow \mathbb{R}$ that is discontinuous for every $x \in [0, 1]$ while $|f|$ is continuous for every $x \in [0, 1]$.

4. (20 points) Show that the equation $x = \cos x$ has a solution on the interval $[0, \frac{\pi}{2}]$. (Hint: Use the Intermediate Value Theorem).

5. (20 points) Show that $f(x) = \frac{1}{x^2}$ is uniformly continuous on $[1, \infty)$ while it is not uniformly continuous on $(0, \infty)$.